Exercise 21

For the following exercises, find the x- or t-intercepts of the polynomial functions.

$$f(x) = x^6 - 2x^4 - 3x^2$$

Solution

To find the x-intercepts, set f(x) = 0 and solve the equation for x.

$$x^{6} - 2x^{4} - 3x^{2} = 0$$

$$x^{2}(x^{4} - 2x^{2} - 3) = 0$$

$$x^{2}(x^{2} - 3)(x^{2} + 1) = 0$$

$$x^{2}(x + \sqrt{3})(x - \sqrt{3})(x^{2} + 1) = 0$$

$$x^{2} = 0 \quad \text{or} \quad x + \sqrt{3} = 0 \quad \text{or} \quad x - \sqrt{3} = 0 \quad \text{or} \quad x^{2} + 1 = 0$$

$$x = 0 \quad \text{or} \quad x = -\sqrt{3} \quad \text{or} \quad x = \sqrt{3} \quad \text{or} \quad x^{2} = -1$$

$$x = 0 \quad \text{or} \quad x = -\sqrt{3} \quad \text{or} \quad x = \sqrt{3} \quad \text{or} \quad (\text{no real soln})$$

Therefore, the x-intercepts are $(-\sqrt{3},0)$ and (0,0) and $(\sqrt{3},0)$.

